MORE ON THE OBSERVATIONAL EQUIVALENCE OF VARIOUS MACROECONOMIC MODELS

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I INTRODUCTION

This paper reinterprets and extends an important note by Sargent(1976b). Sargent showed that any stationary process for output and money could be represented in two forms. The first is consistent with the hypothesis that the past and present levels of money affect output while the second states that only the history of unexpected changes in money influences output. In this paper Sargent's result is extended in two directions. First, it is shown to hold for a more general multivariate system. Second, it is demonstrated that there is a third representation consistent with any stationary multivariate system that includes money and output. In this representation, the only effect of money on output at time t is through the history of the differences beetween money at t and the mathematical expectation of money at t conditional on past information. This representation captures the main implication of a theory according to which the cyclic behaviour of output is due to the presence of contracts written in nominal terms (Fischer (1977),Phelps and Taylor (1977)).

This observational equivalence between the Keynesian, natural rate and "contracting" models would not be very important if the acceptance by the data of restricted versions of one model could be construed as providing empirical support for that model at the expense of the other two. This however is not the case. A restricted version of any of these models is equivalent to restricted versions of the other two. Moreover, the restrictions whose acceptance prompted Leiderman (1980) to say:"money growth appears to affect unemployment in the United States only when this growth is unanticipated" are consistent with the opposite viewpoint. They are consistent with a strong impact of the levels of money on unemployment together with an

extremely successful monetary policy.

This paper further criticizes the tests conducted by Barro (1977, 1981) and Makin (1981). They accept the Keynesian view that the levels of money affect output only when the explanators of money do not themselves explain output. The Employment Act of 1946 suggests that the variables to which money responds are precisely those which influence output. Therefore their tests are likely to reject the Keynesian view. As an alternative to these tests I propose a test of the Keynesian vs. the "new classical" model of Lucas (1972) based on Bayesian considerations. Each model is taken to be a set of equations together with a prior over the parameters of these equations. This prior thus includes all the restrictions across coefficients that the proponents of each model believe in. It is shown that the two models can be tested against each other even when the prior are diffuse as long as money is affected by finitely many lags of money.

The paper proceeds as follows. Section II extends Sargent's (1976) observational equivalance to a multivariate system and interprets this equivalance as casting doubts on the methodology of Barro and Rush (1980), Leiderman (1980) and Mishkin (1980). Section III considers the testing methods of Barro (1977) and Makin (1981) while section IV offers a Bayesian alternative to these tests. Section V extends the observational equivalence argument to models with nominal contracts and section VI presents some concluding remarks.

II REINTERPRETING SARGENT'S OBSERVATIONAL EQUIVALENCE

Let there be three types of variables which move over time. First, there is an index of activity denoted y_t at time t. Then there is a monetary control variable m_t and, finally, there is a vector x_t which includes other variables of interest.

As long as these variables are stationary[1/] their movement over time can be described by the Wold representation:

$$\begin{bmatrix} y_t \\ m_t \\ x_t \end{bmatrix} = \Lambda(L)\begin{bmatrix} a_t \\ b_t \\ c_t \end{bmatrix} = \begin{bmatrix} \alpha_1(L) & \alpha_2(L) & \alpha_3(L) \\ \beta_1(L) & \beta_2(L) & \beta_3(L) \\ \gamma_1(L) & \gamma_2(L) & \gamma_3(L) \end{bmatrix} \begin{bmatrix} a_t \\ b_t \\ c_t \end{bmatrix} \tag{1}$$

where $\Lambda(L)$ is a matrix of polynomials in the lag operator L such that $L^k b_t = b_{t-k}$ and $\alpha_1(L) = \alpha_{10} + \alpha_{11}L + \alpha_{12}L^2 \ldots\ldots$ The variates a_t, b_t and c_t have mean zero, are mutually and serially uncorrelated and have finite variances.

As long as $\Lambda(L)$ is invertible [2/] the evolution of the vector $[y_t, m_t, x_t]$ can be interpreted along Keynesian lines. Here this will mean that the history of the levels of money and other variables affects unemployment and output in a systematic fashion. The coefficients corresponding to this interpretation are obtained by premultiplying both sides of (1) by the inverse of $\Lambda(L)$.

$$\Lambda^{-1}(L)\begin{bmatrix} y_t \\ m_t \\ x_t \end{bmatrix} = \begin{bmatrix} \Phi_1(L) & \Phi_2(L) & \Phi_3(L) \\ \Theta_1(L) & \Theta_2(L) & \Theta_3(L) \\ \Psi_1(L) & \Psi_2(L) & \Psi_3(L) \end{bmatrix} \begin{bmatrix} y_t \\ m_t \\ x_t \end{bmatrix} = \begin{bmatrix} a_t \\ b_t \\ c_t \end{bmatrix} \tag{2}$$

A large number of forms of $\Lambda^{-1}(L)$ are consistent with any given covariance structure among the y's m's and x's. Therefore I will make certain assumptions consistent with Barro's (1977) which make $\Lambda^{-1}(L)$ unique [3/]. These are that $\Phi_{10} = \Theta_{20} = 1$, $\Psi_{30} = I$ and $\Psi_{10} = \Psi_{20} = \Theta_{10} = 0$. Under these assumptions b_t is the innovation in money at t in the sense of Sims (1980). Furthermore b_t corresponds to the difference beetween money at t and its expectation as defined in the work of Barro.

As long as $A(L)$ and $\alpha_1(L)\gamma_3(L)-\alpha_3(L)\gamma_1(L)$ are invertible, (1) can be written as:

$$\begin{aligned} B_1(L)y_t + B_2(L)b_t + B_3(L)x_t &= a_t \\ \Theta_1(L)y_t + \Theta_2(L)m_t + \Theta_3(L)x_t &= b_t \\ \Psi_1(L)y_t + \Psi_2(L)m_t + \Psi_3(L)x_t &= c_t \end{aligned} \qquad (3)$$

where:

$$\begin{aligned} B_1(L) &= \gamma_3(L)/D(L) \\ B_2(L) &= [\alpha_3(L)\gamma_2(L) - \alpha_2(L)\gamma_3(L)]/D(L) \\ B_3(L) &= -\alpha_3(L)/D(L) \\ D(L) &= \alpha_1(L)\gamma_3(L) - \alpha_3(L)\gamma_1(L) \end{aligned}$$

This representation embodies the "natural rate theory". Here, only the unexpected changes in m_t explain movements in y_t. The key question from the point of view of monetary policy is whether changes in the β's will induce changes in the Φ's of (2) or in the B's of (3). Clearly this question cannot be answered in general when only observations from a single monetary regime are available. In the words of Barro (1981): "with no further restrictions imposed on the model, it is impossible to distinguish the system (3) from the system (2)".

Barro seems to think that, if the system (3) is restricted, it can be distinguished from (2). A restricted version of (3) can naturally be tested against an unrestricted version of (3). But, what does this say about (2) ?

Suppose (3) is true. Then, (2) holds with:

$$\Phi_1(L) = B_1(L) + B_2(L)\Theta_1(L)$$

$$\Phi_2(L) = B_2(L)\Theta_2(L) \tag{4}$$

$$\Phi_3(L) = B_3(L) + B_2(L)\Theta_3(L)$$

If a restricted version of (3) is true then (4) implies that a restricted version of (2) is true. The validity of a restricted version of (3) can thus only cast doubts on the Keynesian model if the implied restrictions on (2) are unintuitive from the Keynesian point of view. Instead those restrictions which Barro (1977,1981) and Leiderman (1981) have found to be valid have a very appealing Keynesian interpretation. I will call the restrictions imposed by these authors as well as by Mishkin (1980) "nondirectness" restrictions. They imply that, in (3), those variables which explain the history of money up to m_t have no direct impact on y_t. Nondirectness is equivalent to $\Theta_3(L) \neq 0$, $B_3(L) = 0$ and $B_1(L) = 1$ 4/. Nondirectness restricts (2). It implies that:

$$\frac{\Phi_2(L)}{\Phi_3(L)} = \frac{\Theta_2(L)}{\Theta_3(L)} \qquad \text{and} \qquad \frac{\Phi_1(L) - 1}{\Phi_2(L)} = \frac{\Theta_1(L)}{\Theta_2(L)}$$

Once due account is taken of the response of m_t to changes in x and lagged y's, these changes have no effect on y. It is, in fact, this restricted version of (2) which the authors cited at the beginning of this section have used in their empirical work.

Suppose that the first equation of (2) is invariant to changes in the monetary rule. Suppose further that, consistent with the Employment Act of 1946 the monetary authority seeks only to stabilize output and bring y_t as close as possible to y_t^*. If the Fed has a quadratic objective function it should follow a monetary rule such that:

$$E[\Phi_2(L)m_t + \Phi_3(L)x_t + (\Phi_1(L) - 1)y_t + y_t^*] = 0 \tag{5}$$

where E is the operator which takes expectations conditional on the information available to the Fed before it sets m_t.

If, as in Barro (1977) the monetary authority observes c_t but not a_t when it picks m_t and if, additionally, the Fed's plan is carried out with error, output will follow the process:

$$y_t - y_t^* + \Phi_{20}b_t = a_t \tag{6}$$

where b_t is the error made by the monetary authority at t.If y_t^* is constant (6) satisfies the nondirectness restrictions. Furthermore (6) has the property that $B_2(L) = \Phi_{20}$. McCallum (1979a) notes that for a model to be truly consistent with the model of Lucas (1972) $B_2(L)$ should indeed be of order zero.[5/]

Barro (1977) rejecs this hypothesis. McCallum (1970a) argues that this rejection is probably due to Barro's omission of certain state variables (like past values of y, inventories and the capital stock) as explanators of y_t. These variables would be important if they were a part of y_t*. Therefore, the equation McCallum proposes is identical to (6). In other words the hypothesis that only the latest innovation in money affects output is observationally equivalent to the hypothesis that the levels of money and of its explanators influence output while the monetary authority is following an optimal stabilization program with error.[6/]

On the other hand there is another Keynesian explanation for the dependence of $B_2(L)$ on the low powers of L. Suppose that the monetary authority either discovers the true value of the money stock with a lag or simply reacts slowly to this true value. Then, in carrying out its monetary policy it may substitute the target value of m_{t-k} for the true value of m_{t-k} in (5). In this case output will also be a function of b_{t-k} in addition to being a function of b_t.

A Keynesian should also not be surprised to find that (3) with the assumption

of nondirectness holds across different policy regimes. After all, the monetary authority should change its feedback rule only when the Φ's change and should always ensure that (6) holds. This renders the tests carried out by Sargent and Neftci (1978) of somewhat dubious value in differentiating the Keynesian view from the view that monetary policy is neutral in the short run.

The preceding discussion suggests that acceptance of a restricted version of (3) does not constitute evidence particularly favorable to the natural rate hypothesis. Moreover, rejection of the nondirectness restrictions does not cast strong doubts on the natural rate hypothesis. Suppose that some lagged variable affects y_t. Then, if the Federal Reserve even thinks that it can stabilize output it will react to this variable. It follows that money and output will have common explanators and nondirectness will be rejected even if money is neutral in the short run.

III THE MAINTAINED HYPOTHESIS APPROACH

This section discusses the tests of the neutrality of monetary policy conducted by Barro (1977, 1981) and Makin (1981). In these papers the hypothesis of nondirectness is maintained in both the new classical and the Keynesian versions of the model. In other words current output is explained either only by a finite set of lagged monetary innovations or only by a finite set of lagged levels of money. I will argue the latter is an unfair representation of the Keynesian model.

The procedure consists of comparing the fit of (3) with the assumptions $B_1(L) = 1$ and $B_3(L) = 0$ with the fit of (2) with the assumptions $\Phi_1(L) = 1$ and $\Phi_3(L) = 0$. These two systems are nonnested. However they can be nested in a composite system whose first equation is:

$$y_t + N_1(L)b_t + N_2(L)m_t = a_t \tag{7}$$

The classical hypothesis together with nondirectness is accepted if $N_2(L)$ is not significantly different from zero. Instead, the Keynesian view together with nondirectness is accepted if $N_1(L)$ is not significantly different from zero. [7] At first glance this appears to be a standard nonnested

test. There is a maintained hypothesis, namely nondirectness, and two alternative hypotheses which compete as explanators of the data. However, this particular maintained hypothesis has two undesirable properties.

First, it is theoretically incompatible with the Keynesian viewpoint. The Employment Act of 1946 forces the government to try to stabilize output. If money is indeed capable of regulating output then one would expect the variables the Fed responds to to be variables which affect output. Therefore nondirectness cannot fairly be appended to the Keynesian model.

Second, it is easy to identify alternative maintained hypothesis of the same type which are more favorable to either of the two models being tested. Typically, an investigator will maintain hypotheses, such as linearity, whose replacement by hypotheses of the same type ,such as loglinearity, will affect his tests in an unpredictable way. Here, instead, once a system like (2) has been estimated an investigator knows the direct effects which, when postulated, eliminate the explanatory power of the b's or of the m's in (7).

Finally,it must be noted that the system whose first equation is (7) is a restricted version of (2). These restrictions can be tested. They amount to testing whether the history of levels and innovations of money is sufficient to explain output or whether lagged values of y and/or current and lagged values of x also affect output. Eq. (7) therefore embodies a weak form of the nondirectness postulate for either models (2) or (3). This test has been discussed by Abel and Mishkin (1981) and perfomed by Leiderman (1980) and Mishkin (1980). These authors have called it a "test for rationality". This name has the following origin: First, these authors do not consider the possibility that direct effects may exist. Second, they deem "rational" situations in which only the expected and unexpected components of money affect output. Then (7) can be rewritten as:

$$y_t + [N_1(L) + N_2(L)]b_t + N_2(L)\hat{m}_t = a_t$$

where $\hat{m}_t$ is equal to $(m_t - b_t)$ and is the value of m_t that is expected by rational agents before they observe m_t.

IV A BAYESIAN VIEWPOINT

The tests using the maintained hypothesis of nondirectness would be appropiate if all economists believed that the direct effects are zero. However this is a belief that Keynesians are unlikely to share. A test which imposes fewer restrictions on the beliefs of economists is proposed in this section. Let both Keynesians and people who believe in the neutrality of short run monetary policy have prior beliefs over the parameters of (2) and (3).Then, as long as the resulting marginal densities of the observations don't coincide , the two models together with their priors can be tested using standard Bayesian techniques (see Zellner (1971) or Leamer (1978) for expositions).Let C be the new classical model represented by (3) and K be Keynesian model represnted by (2). Then the relative posterior probabilities of the two models can be written as:

$$\frac{P(K|Y)}{P(C|Y)} = \frac{P(Y|K)}{P(Y|C)} \quad \frac{P(K)}{P(C)} \tag{8}$$

where Y is the vector of observations and $P(K|Y)$is the posterior probability of model K given the observations Y. The first term in brackets is the "Bayes factor" , it summarizes the effect of the data on the relative believability of the two models. The second term in brackets is the prior odds ratio.

The data can only help establish the odds in favor of each model if $P(Y|K)$ is in general different from $P(Y|C)$. If, instead, these probabilities always coincide then the models are observationally equivalent.

If a_t, b_t and c_t are normal variates with zero mean and variances σ_a^2, σ_b^2 and σ_c^2 then $P(Y|K)$ can be written as follows.

$$P(Y|K) = \int_{-\infty}^{\infty} d\Phi d\Theta d\Psi d\sigma_a d\sigma_b d\sigma_c (2\pi\sigma_a^2\sigma_b^2\sigma_c^2)^{-T/2} \exp[\sum_{t=0}^{T} \frac{(\Phi_1(L)y_t + \Phi_2(L)m_t + \Phi_3(L)x_t)^2}{\sigma_a^2}$$

$$+ \sum_{t=0}^{T} \frac{(\Theta_1(L)y_t + \Theta_2(L)m_t + \Theta_3(L)x_t)^2}{\sigma_b^2} + \sum_{t=0}^{T} \frac{(\Psi_1(L)y_t + \Psi_2(L)m_t + \Psi_3(L)x_t)^2}{\sigma_c^2}]$$

$$P_K(\Phi,\Theta,\Psi,\sigma_a,\sigma_b,\sigma_c) \tag{9}$$

while $P(Y|C)$ can be written as:

$$P(Y|C) = \int_{-\infty}^{\infty} dBd\Theta d\Psi d\sigma_a d\sigma_b d\sigma_c (2\pi\sigma_a^2\sigma_b^2\sigma_c^2)^{-T/2} \exp[\sum_{t=0}^{T} \frac{(\Theta_1(L)y_t + \Theta_2(L)m_t + \Theta_3(L)x_t)^2}{\sigma_b^2}$$

$$+ \sum_{t=0}^{T} \frac{(B_1(L)y_t + B_2(L)(\Theta_1(L)y_t + \Theta_2(L)m_t + \Theta_3(L)x_t) + B_3(L)x_t)^2}{\sigma_a^2}$$

$$+ \sum_{t=0}^{T} \frac{(\Psi_1(L)y_t + \Psi_2(L)m_t + \Psi_3(L)x_t)^2}{\sigma_c^2}] \; P_C(B,\Theta,\Psi,\sigma_a,\sigma_b,\sigma_c) \tag{10}$$

where $P_K(\Phi,\Theta,\Psi,\sigma_a,\sigma_b,\sigma_c)$ and $P_C(B,\Theta,\Psi,\sigma_a,\sigma_b,\sigma_c)$ are the priors over the parameters which correspond to models K and C respectively. For many choices of P_K and P_C the two models will not be observationally equivalent. If the resulting ratio of $P(Y|K)$ over $P(Y|C)$ is very large one will be able to reject the new classical model together with P_C in favor of the Keynesian model together with P_K.[8]

The main difficulty with applying such a Bayesian analysis to this problem is to arrive at priors which represent the two models while ensuring

that they are not observationally equivalent. However, under certain weak conditions, even uninformative priors will distinguish the two models. In particular, let the two priors be given by:

$$P_K(\Phi,\Theta,\Psi,\sigma_a,\sigma_b,\sigma_c) = 1/(\sigma_a\sigma_b\sigma_c)$$
$$P_C(B,\Theta,\Psi,\sigma_a,\sigma_b,\sigma_c) = 1/(\sigma_a\sigma_b\sigma_c) \tag{11}$$

where the standard errors are only allowed to be positive.[9/] The priors given by (11) are much more flexible than those which are equivalent to the maintained hypothesis approach. However, it must be noted that they do not include as part of the Keynesian model an optimizing Federal Reserve like the one considered in section II.

With these priors, as long as $\Theta_2(L)$ is different from one and has finite order, there is no change of variables which produces (10) from (9). This is only a necessary condition for (9) and (10) to be different. However, I conjecture that, for most realizations of the vectors (y_t,m_t,x_t) this condition will also be sufficient to differentiate (9) from (10).

The two models could be differentiated if the order of the Θ and Ψ polynomials were different in (9) and (10). Still, the length of the lags that explain money and x is not an area in which supporters of K and of C disagree. Therefore, I will assume that the order of $\Theta(L)$ and of $\Psi(L)$ is the same in (9) and (10). Then, a change of variables exists which converts (9) into (10) if there exists a transformation of Φ into B which when applied to (9) produces (10). Such a transformation would have the properties given by (4).

I will assume that any variable which affects y_t indirectly through

money is also able to affect it directly in the C model. This relaxation of the nondirectness assumption ensures that the orders of $B_1(L)$ and $B_2(L)$ are at least as big as the orders of $B_2(L)\Theta_1(L)$ and $B_2(L)\Theta_3(L)$ respectively. On the other hand, let the order of $B_2(L)$ be k. The Jacobian of the transformation then has the following form:

$$\left|\frac{d\Phi}{dB}\right| = \begin{vmatrix} d\Phi_{1i}/dB_{1j} & d\Phi_{1i}/dB_{2j} & d\Phi_{1i}/dB_{3j} \\ d\Phi'_{2i}/dB_{1j} & d\Phi'_{2i}/dB_{2j} & d\Phi'_{2i}/dB_{3j} \\ d\Phi''_{2i}/dB_{1j} & d\Phi''_{2i}/dB_{2j} & d\Phi''_{2i}/dB_{3j} \\ d\Phi_{3i}/dB_{1j} & d\Phi_{3i}/dB_{2j} & d\Phi_{3i}/dB_{3j} \end{vmatrix}$$

$$= \begin{vmatrix} I & DZ & 0 \\ 0 & LT & 0 \\ 0 & DZ & 0 \\ 0 & DZ & I \end{vmatrix} \qquad (12)$$

where Φ'_2 contains the first k elements of Φ_2 and Φ''_2 contains the remaining elements. The identity matrix is denoted by I while the DZ matrices have, in general, nonzero elements troughout. LT is a lower triangular matrix with ones on the diagonal which are due to the equality of Θ_{20} and one.

If the order of $\Theta_2(L)$ is bigger than zero and finite no order for $\Phi_2(L)$ generates the desired transformation. Letting $\Phi_2(L)$ have order bigger than k leads to a singular Jacobian. Instead, if k is the order of $\Phi_2(L)$ the C model includes more regressors than the corresponding K model.

If, on the contrary, the order of $\Theta_2(L)$ is zero and money is not explained by its own past then $\Phi_2(L)$ need only be of order k for both models to include the same explanatory variables. Moreover the determinant in (12) is one and therefore the models are indistinguishable in this case.[10/]

So, when K and C allow all "direct" effects the explanators of money other than its own past cannot by themselves determine which model is better. Instead, when money depends on its own history the C model predicts that the impact of lags of m_t on y_t is related to the impact of more recent m's on y_t. The K model makes no such prediction. This difference allows one to compare the fit of the two models.

V EXTENSION TO MODELS WITH NOMINAL CONTRACTS

An important class of models due to Phelps and Taylor (1977) and Fischer (1977) has the property that the only impact of money on output at t is due to the history of the differences beetween m_t and the past expecattion of money at t. In these models workers or firms sign contracts at t-k for goods or services at t whose price is determined in nominal terms at t-k. This leads to a system of equations of the form:

$$
\begin{aligned}
&K_1(L)y_t + \sum_{k=0}^{\infty} K_{2k}(m_t - m_{t-k/t}) + K_3(L)x_t = a_t \\
&\Theta_1(L)y_t + \Theta_2(L)m_t + \Theta_3(L)x_t = b_t \qquad (13) \\
&\Psi_1(L)y_t + \Psi_2(L)m_t + \Psi_3(L)x_t = c_t
\end{aligned}
$$

where $m_{t-k/t}$ is the mathematical expectation of money at t conditional on all information available at time t-k.

I now establish that any system like (1) can be arbitrarily well approximated by a system of the form of (13) 11/. In other words, unrestricted systems of the form of (13) can never be rejected by finite data.

The second two equations of (13) follow from the invertibility of Λ. I will show that K's such that the first equation holds exist for processes which are arbitrarily close approximations to (1).

By the Wiener-Kolmogorov prediction formula $m_{t-k/t}$ can be written as:

$$m_{t-k/t} = \sum_{i=k}^{\infty}(\beta_{1i}a_{t-i} + \beta_{2i}b_{t-i} + \beta_{3i}c_{t-i})$$

Hence:

$$m_t - m_{t-k/t} = \sum_{i=0}^{k-1}(\beta_{1i}a_{t-i} + \beta_{2i}b_{t-i} + \beta_{3i}c_{t-i}) \quad k=1,2,3\ldots\ldots \tag{14}$$

The set of equations (14) is equivalent to the second equation of (1). Hence linear combinations of (14) with the first and third equations of (1) are valid. Consider the following linear combination:

$$K_1(L)[\alpha_1(L)a_t+\alpha_2(L)b_t+\alpha_3(L)c_t]+K_3(L)[\gamma_1(L)a_t+\gamma_2(L)b_t+\gamma_3(L)c_t]+ \sum_{j=1}^{\infty} K_{2j}[\sum_{i=0}^{j-1}(\beta_{1i}a_{t-i}+\beta_{2i}b_{t-i}+\beta_{3i}c_{t-i})] = a_t \tag{15}$$

If K's can be found such that (15) holds for each set of α's, β's and γ's (13) is a valid representation of (1). Changing the order of summation in (15) one obtains:

$$[K_1(L)\alpha_1(L) + K_3(L)\gamma_1(L)]a_t+[K_1(L)\alpha_2(L)+K_3(L)\gamma_2(L)]b_t+[K_1(L)\alpha_3(L)+K_3(L)\gamma_3(L)]c_t + \sum_{i=0}^{\infty}(\beta_{1i}a_{t-i}+\beta_{2i}b_{t-i}+\beta_{3i}c_{t-i})(\sum_{j=i+1}^{\infty}K_{2j}) = a_t$$

The K's must therefore satisfy:

$$[K_1(L)\alpha_1(L)+K_3(L)\gamma_1(L)]a_t + \sum_{i=0}^{\infty} \beta_{1i} a_{t-i} \sum_{j=i+1}^{\infty} K_{2j} = a_t$$

$$[K_1(L)\alpha_2(L)+K_3(L)\gamma_2(L)]b_t + \sum_{i=0}^{\infty} \beta_{2i} b_{t-i} \sum_{j=i+1}^{\infty} K_{2j} = 0$$

$$[K_1(L)\alpha_3(L)+K_3(L)\gamma_3(L)]c_t + \sum_{i=0}^{\infty} \beta_{3i} c_{t-i} \sum_{j=i+1}^{\infty} K_{2j} = 0$$

Define $\tilde{\beta}_1(L), \tilde{\beta}_2(L)$ and $\tilde{\beta}_3(L)$ as follows

$$\tilde{\beta}_{si} = \beta_{si} \sum_{j=i+1}^{\infty} K_{2j} \qquad s = 1,2,3$$

Then the above system can be rewritten as:

$$K_1(L)\alpha_1(L) + K_3(L)\gamma_1(L) + \tilde{\beta}_1(L) = 1$$

$$K_1(L)\alpha_2(L) + K_3(L)\gamma_2(L) + \tilde{\beta}_2(L) = 0 \tag{16}$$

$$K_1(L)\alpha_3(L) + K_3(L)\gamma_3(L) + \tilde{\beta}_3(L) = 0$$

As long as $\alpha_1(L), \alpha_2(L), \alpha_3(L)\gamma_2(L)-\alpha_2(L)\gamma_3(L)$ and $\alpha_1(L)\gamma_2(L)-\alpha_2(L)\gamma_1(L)$ are invertible one can eliminate $K_1(L)$ and $K_3(L)$ from (16) and obtain:

$$\tilde{\beta}_1(L)[\alpha_3(L)\gamma_2(L)-\alpha_2(L)\gamma_3(L)]+\tilde{\beta}_2(L)[\alpha_1(L)\gamma_3(L)-\alpha_3(L)\gamma_1(L)] +$$

$$\tilde{\beta}_3(L)[\gamma_1(L)\alpha_2(L)-\gamma_2(L)\alpha_1(L)] = \alpha_3(L)\gamma_2(L) - \alpha_2(L)\gamma_3(L) \tag{17}$$

Equation (17) can be solved recursively for the powers of L starting from zero:

$$[\sum_{j=1}^{\infty} K_{2j}][\beta_{10}(\alpha_{30}\gamma_{20}-\alpha_{20}\gamma_{30})+\beta_{20}(\alpha_{10}\gamma_{30}-\alpha_{30}\gamma_{10})+\beta_{30}(\gamma_{10}\alpha_{20}-\gamma_{20}\alpha_{10})]=\alpha_{30}\gamma_{20}-\alpha_{20}\gamma_{30}$$

This equation can only be solved if $(\beta_{10},\beta_{20},\beta_{30})$ is not a linear combination of $(\alpha_{10},\alpha_{20},\alpha_{30})$ and $(\gamma_{10},\gamma_{20},\gamma_{30})$. More generally K_{2i} can be obtained from:

$$K_{2i}[\beta_{1i}(\alpha_{30}\gamma_{20}-\alpha_{20}\gamma_{30})+\beta_{2i}(\alpha_{10}\gamma_{30}-\alpha_{30}\gamma_{10})+\beta_{3i}(\gamma_{10}\alpha_{20}-\gamma_{20}\alpha_{10})]= Z_i$$

where Z_i depends on the α's, β's and γ's, on the $\sum_{j=1}^{\infty} K_{2j}$ and on K_{2s} for s smaller than i. Therefore the sequence K_2 can be obtained as long as the vectors $(\beta_{1i},\beta_{2i},\beta_{3i})$ are not linear combinations of $(\alpha_{10},\alpha_{20},\alpha_{30})$ and $(\gamma_{10},\gamma_{20},\gamma_{30})$. Note however that if the above condition is violated, K_2's can be obtained for arbitrarily close approximations of (1). $K_1(L)$ and $K_3(L)$ can then be computed using the first two equations of (16).

This establishes that a very general "contracting" model can never be rejected by finite data. However, restricted versions of (13) like those of Taylor (1980) and Sheffrin (1978) can be rejected by such data.[12/] It must be kept in mind, nonetheless, that these restricted versions of (13) are also consistent with restricted versions of (1) and hence of (2) and (3). Therefore acceptance of these restrictions cannot in general be interpreted as acceptance of the contracting model as opposed to the Keynesian and new classical models. Such acceptance requires that the restrictions which the data do not reject be unlikely to hold if the Keynesian or natural rate views were correct. This type of analysis can be carried out using the Bayesian framework of section IV.

VI CONCLUSIONS

This paper argues that one of the central problems of empirical macroeconomics is the richness of models (or interpretations) consistent with any set of covariances beetween variables which move over time. It is not just that unrestricted versions of models with nominal contracts, models in which the level of money affects economic activity and models in which only the unanticipated component of monetary changes influences output can account for any covariance stationary system. What is more important is that rest-

ricted versions of any of these models are equivalent to restricted versions of the other two. Therefore acceptance of the restricted version of a model does not, per se, provide much support for the model. In particular the restrictions accepted by Barro (1977,1981) and Leiderman (1980) are consistent with a Keynesian model in which the government is carrying out an outstanding monetary policy. Only when the restrictions that the data do not reject are intuitively appealing when interpreted in the light of one model and unappealing in the light of the others can one say that the former is preferred by the data. Bayesian methods can be used to discover which model together with the beliefs about its parameters that go with it receives most support from the data.

FOOTNOTES

1/ The arguments below obviously apply also to transformations of these variables which are stationary.

2/ If $A(L)$ is not invertible, the evolution of (y_t, m_t, x_t) can be arbitrarily well approximated by the model below.

3/ Buiter (1980) argues that these assumptions are not as innocent as they appear. Mishkin(1980) assumes instead that : $\Theta_{10} = \Theta_{30} = \Phi_{30} = 0$. The analysis below is valid also under Mishkin's assumptions.

4/ Barro (1977) allows certain variables which do not explain money to explain unemployment. These, however, do not contribute to the empirical distinction beetween (2) and (3) and will not be considered here.

5/ It is worth noting that McCallum(1979a)does not believe that a scenario in which output depends only on its own past and on the current monetary innovation is consistent with a Keynesian view of the world. This paper refutes that belief since y_t^*, the target level of output, may well depend on past levels of y if there adjustment costs.

6/ This argument is similar to the argument made by Goldfeld and Blinder (1972) in the context of deterministic models of fical policy. These authors show that if fiscal policy is chosen optimally without error, fiscal policy variables will tend to be poor explanators of output.

7/ In this framework it is,of course, possible to either accept both or reject both of these hypotheses.

8/ Note that the test discussed in section III is similar to this one with priors which attribute unit probability to $\Phi_3(L)=B_3(L)=0$ and to $\Phi_1(L)=B_1(L)=1$

9/ These priors require numerical integration to obtain (9) and (10).

10/ When the Φ's go between minus and plus infinity so do the B's by (4). Therefore the limits of integration do not help distinguish these models.

11/ McCallum (1979b) incorrectly states that this proposition has been proved by Sargent (1976b).

12/ These authors restrict the order of $K_1(L)$, $K_2(L)$ and $K_3(L)$. Sheffrin (1978) forces $K_1(L)$ to be of order one, $K_3(L)$ to be equal to zero and $K_2(L)$ to be finite. Taylor (1980) derives restrictions on the orders of these polynomials by restricting all contracts to have the same finite length and by assuming that a proportion 1/n is renewed each period.

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